Shortening Hamming Codes to Better Correct 2-Bit Errors

Cannon McIntosh (Coastal Carolina University) & Willy Kim (Florida State University)

Mentor: Laura Monroe

Motivation

- 2-bit errors are becoming more frequent
- DECTED codes cost more memory and time
- want SECDED 2-bit error correction
  - need weight-2 vector distribution
  - need contextual information from application
- wish to construct codes that do this more effectively
  - Use coding theory, combinatorics, and geometry

Method

- find all possible weight-2 correction vectors
  - weight-2 vectors in same coset as received vector
- XOR received vector with each correction vector
  - this generates short list of all possible solutions
- use contextual information to choose best candidate

Codes in Common Use

- Hamming and Hsiao are common SECDED codes
  - poor for 2-bit error correction
  - high variance in number of correction vectors
  - high average number of correction vectors
- Can we do better?

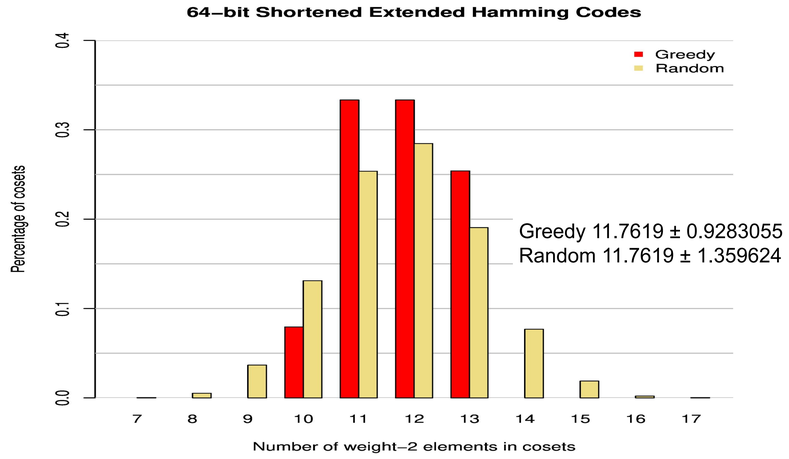
Shortening

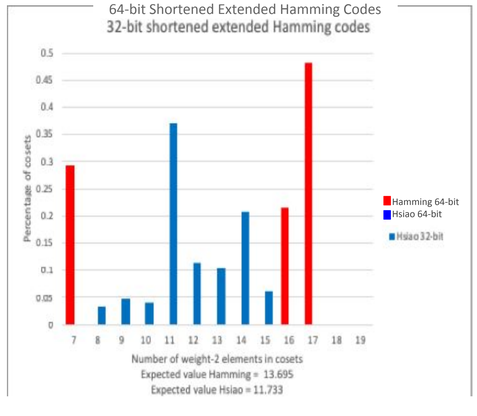
- omit codewords with 1's in selected bit-position
  - reduces dimension by 1
  - continue until dimension is a power of 2
- Want a code with lower variance and expected value

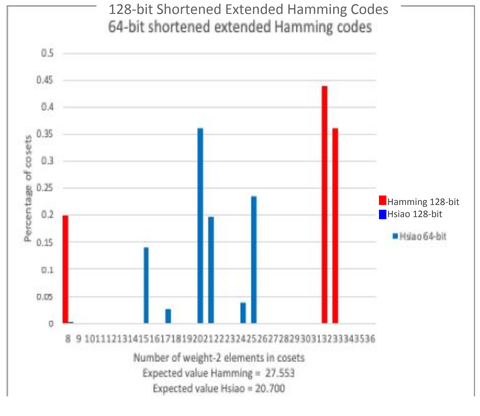
Greedy Shortening

- among cosets with the least weight-2 vectors
  - shorten position if it removes no weight-2 vectors
- else, among cosets with the most weight-2 vectors
  - shorten to maximize weight-2 vectors removed

LA-UR-19-27162

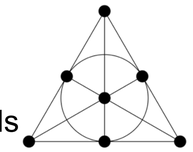






Block Designs

- Extended Hamming codes are rich in block designs
- **1-(2^r, 4, 2^{r-1})**
- **2-(2^r, 4, 2^{r-1}-1)**
- **3-(2^r, 4, 1)**
- useful tool to construct tighter bounds



Constructive Bounds

- code of length 2^r
  - number of weight-2 vectors in a coset
    - lower bound $r+1$
    - upper bound $2^{r-2} + \lfloor \frac{r+1}{2} \rfloor$

32-bit Code After 8 Shortenings

- any 8 columns contain a weight-4 codeword
- some coset has 2 double hit weight-2 vectors
- that coset has 10 weight-2 vectors left

- $\binom{8}{2}$ = 28 weight-2 vectors double hit
- 31 cosets in total
- at least 3 cosets with only single hit vectors
- those 3 cosets have 8 vectors left

Future Work

- upper bound of shortenings before divergence
- optimal shortening method
- extend to non-SECDED codes

Collaborators and References

- Thomas Hoffman, Ogul Duncan, William Jones, Nathan DeBardeleben, Vanessa Job
- M. Gottscho, C. Schoeny, L. Dolecek, and P. Gupta, "Software-defined error-correcting codes," in 46th Annual IEEE/IFIP International Conference on Dependable Systems and Networks Workshop (DSN- W), June 2016, pp. 276–282.
- A. Poulos, D. Wallace, R. Robey, **L. Monroe**, V. Job, S.Blanchard, W. Jones and N. DeBardeleben. "Improving Application Resilience by Extending Error Correction with Contextual Information". IEEE Fault Tolerance for HPC at eXtreme Scale (FTXS) Workshop, Supercomputing 2018, November 2018.

COASTAL CAROLINA UNIVERSITY

FLORIDA STATE UNIVERSITY

Ultrascale Systems Research Center

Los Alamos NATIONAL LABORATORY EST.1943