Some exact statistical results for binary mixing and reaction in variable density turbulence

J. R. Ristorcelli
Los Alamos National Laboratory, Los Alamos, NM 87545

Abstract

We report a number of exact statistical results on binary active scalar mixing and reaction in variable density turbulence. The results are relevant to isobaric isothermal material mixing, isobaric thermal mixing and turbulent combustion for which a progress variable is used. We derive expressions relating various second order moments of the mass fraction, specific volume and density fields. We highlight the central role of the density specific volume covariance $\langle \rho v \rangle$ as a measure of the difference between Favre and Reynolds means and as a key quantity with considerable dynamical significance linking several second order statistics. For laboratory experiments we have developed exact relations between the Reynolds scalar variance $\langle c^2 \rangle$ its Favre analog $\widetilde{c''^2}$, and various second moments including $\langle \rho v \rangle$. We invoke a binary one step reaction as a metric to assess the state of the mixing. The mean reaction rate in variable density turbulent mixing can be expressed, in closed form, using the first order Favre mean variables and the Reynolds averaged density variance, $\langle \rho^2 \rangle$. The normalized density variance, $\langle \rho^2 \rangle$ plays a role as the mix metric in a way analogous to the normalized mass faction variance $\langle y_1^2 \rangle$ does in constant density turbulence. The use of the normalized Favre variance of the mass fraction, $\widetilde{c''^2} = \langle \rho^* y_1''^2 \rangle / \bar{\rho}$, as a mix and reaction metric is not theoretically justified in variable density turbulence. We document a novel derivation of an expression for $\langle \rho^2 \rangle$ in terms of a rational function of $\langle \rho v \rangle$ that avoids recourse to Taylor series that are slow to converge for large density ratios. We have derived exact results relating several other second and third order moments and see many relations between odd and even order moments demonstrating a natural and inherent skewness in the mixing process of an active scalar in variable density turbulence.

Keywords: Favre averages, mean turbulent reaction rates, variable density turbulence

LANL Report: LA-UR-14-25224